Quiz 4 solutions

TAOCO has just hired you as an engineering intern. They give you the task of determining whether it is worthwhile to replace their current production equipment with a new system that can increase monthly production rate by a factor of 10. The current system can produce units of product at rates (r) between 500 and 2,000 units per month. The new system costs $2 million and has handle production rates (r) between 500 and 20,000 units per month.

**General approach to solution:**

Since you are asked to find the production rate at which max, profit occurs, you need to determine a profit function and find the maximum value. Finding the maximum value will involve finding the root of the derivative of the profit function, which is the production rate.



**Part A (current equipment, 5 points)**

Raw material cost per unit ($/unit) = exp(5-0.0023\*r)+20

where r is the monthly production rate (units/month)

You may assume these raw material costs are the only expenses for the process.



This model provides the cost per unit. To get the total expenses, need to multiply this by the number of units produced:

Total cost ($/month)= (cost/unit)\*(units/month) =

[exp(5-0.0023\*r)+20]\*r

The sales dept. has determined the total monthly income produced by selling the product.



20000\*r/(10+0.1\*r) ($/month)

Profit = Income - Expenses

Profit ($/month) =

[20000\*r/(10+0.1\*r)] – [exp(5-0.0023\*r)+20]\*r



Looking at the profit model, the maximum looks to be about $155,000/month at a production rate of around 1500 units/month.

To find the maximum point of this profit curve, find the root of the derivative.

The derivative of this function, dProfit(r)/dr =

Finding the root of this function using the Newton-Raphson method



|  |  |  |
| --- | --- | --- |
| part A |  |  |
| x | f(x) | f'(x) |
| 500 | 42.60452 | -0.093313747 |
| 956.5728 | 17.64878 | -0.04148074 |
| 1382.042 | 2.569751 | -0.029041664 |
| 1470.527 | 0.119408 | -0.026354521 |
| 1475.058 | 0.000307 | -0.026219094 |
| 1475.07 | 2.05E-09 | -0.026218744 |

Root=1475 units/month

So this is the production rate at which the profit is maximized.

Plugging this into the profit equation: Max profit = $150,400 per month

**Part B. (new equipment, 5 points)** *Same method of solution*

The purchasing dept. has been able to negotiate a lower raw materials cost model based on the higher potential raw materials usage:

Cost per unit ($/unit) = exp(5-0.0023\*r)+10

The sales income model is not affected by the new equipment capacity.

Same analysis as part A, using new expenses model.



Total cost ($/month)= cost/unit\*units/month = [exp(5-0.0023\*r)+10]\*r

Monthly income = 20000\*r/(10+0.1\*r)

Profit = income – expenses

Profit ($/month) = [20000\*r/(10+0.1\*r)] – [exp(5-0.0023\*r)+10]\*r



Expanding the model to see the maximum more easily:

So by visual inspection, the maximum profit is about $170,000 and occurs at a production rate of about 2000 units/month.

Taking the derivative of the profit function



So our task is to find the root of this function.

To do this, we need to take the derivative of this function.





Use this function and its derivative in the Newton-Raphson method to find the root at the maximum profit.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| part B |  |  |  |  |
| x | f(x) | f'(x) | b | next x |
| 500 | 52.60452 | -0.093313747 | 99.26139 | 1063.738 |
| 1063.738 | 23.35779 | -0.038580118 | 64.39694 | 1669.174 |
| 1669.174 | 5.454514 | -0.02072885 | 40.05458 | 1932.31 |
| 1932.31 | 0.846188 | -0.014565072 | 28.99043 | 1990.408 |
| 1990.408 | 0.033591 | -0.013421653 | 26.74815 | 1992.91 |
| 1992.91 | 5.95E-05 | -0.013374135 | 26.65351 | 1992.915 |
| 1992.915 | 1.88E-10 | -0.01337405 | 26.65334 | 1992.915 |

So for the new equipment/process, the maximum profit occurs at a unit production rate of 1993 units/month. Plugging this into the profit model, the maximum profit is $167,500 per month.

Thinking about this, it makes sense. The sales/income is not affected by the change in equipment other than if the production rate increases. The change mainly affects the expenses, which are decreased (see models below). Profits increase due to more units being produced/sold and lower expenses.





**Part C. (Should the company purchase the new equipment?, 5 points)**

Our calculations show that the new equipment will produce a higher rate of profit ($/month) than the current system. The rate of production increased about 35% and profits increased about 11%. This should cause you concern, since the new equipment capacity is nearly 10-fold higher than current production rates. You might want consider the question of how much profits might increase if production is doubled or increased 5-fold?

First, let’s answer the assigned question of whether buying the new equipment is justified by payback time.

One criterion that can be used to make the decision of whether to purchase the new equipment is the payback time, i.e. how many months it will take to pay back the cost of the new equipment based on the increase in profitability (how much more profit the new system makes vs. the old system). Generally, companies would like to have payback times on the order of 1 to 3 years (the shorter the better!).

The maximum profits for the current and new equipment are $150,400 per month and $167,500 per month, respectively. Hence, by using the new equipment, the increased profit per month is $17,100. If this increased profit is used to pay off the cost of the new equipment, it will take

$2,000,000/($17,100/month)=117 months=approx. 10 years

Based on the desired payback time, it does not make sense to purchase the new equipment.

Now in your recommendation, you should do a bit more thinking/analysis of the situation. The main benefit of the new equipment is to provide much greater production capacity. Under what circumstances might having much greater production capacity benefit the company?



Looking at the raw materials cost model, it is clear that the unit cost has basically bottomed out at the current production rates (approx. 2000 units/month). Hence, with increased production, the total expenses will rise linearly with production rate. So to increase profits, the income must at least rise as fast as the expenses.

However, looking at the current income model, as production increases beyond about 5000 units/month, the income flattens out. This implies that the market is basically close to saturation. So unless the total sales/market can be increased by some means, it makes no sense to buy the new equipment to increase production.



Using our models, we can calculate income and profitability if the production rate were to increase from 2000 to 20,000 units per month. Calculating the change in income between 20,000 units/month and 2000 units/month shows a net increase of about $900. At the same time, expenses go up linearly, $200,000 vs. $22,980.

So not surprisingly, the profits actually decrease and we are losing about $1000/month at this production level. We can also see this easily by looking at the profit model.



So your recommendation should be not to purchase the new equipment and further more, unless the market becomes much larger, it does not make sense to consider any expansion of production. Certainly, do not try to increase profits by cutting product price to try to increase sales, as that will only decrease profits since the market appears to be saturated.